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Local Cosmic Strings from Pseudo-Anomalous $U(1)$

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Local cosmic strings solutions are introduced in a model with a peudo-anomalous $U(1)$ gauge symmetry. Such a symmetry is present in many superstring compactification models. The coupling of those strings with the axion necessary in order to cancel the anomalies does not prevent them from being local, even though their energy per unit length is found to diverge logarithmically. I discuss briefly the formation of such strings and the phenomenological constraints that apply to their parameters.

INTRODUCTION

I report here work done in collaboration with P. Binétruy and P. Peter [1] on cosmic strings in models with a pseudo-anomalous $U(1)$ symmetry. Such a symmetry arises generically in a large class of superstring compactification models as a remnant of the Green–Schwarz [6] mechanism of anomaly cancellation in the underlying 10-dimensional supergravity. In a bottom top approach, interest in such models has recently been renewed in the framework of horizontal symmetries trying to explain the hierachies in the quark and lepton spectra [25].

It is generically well known that cosmic strings may form in the early universe in the breaking of a $U(1)$ symmetry [7, 11].[2] This is also true when the symmetry is pseudo-anomalous [4, 9, 10, 27]. However, because of their being coupled to the axion field, such strings were thought to be of the global kind. We show that there exists a possibility that (at least some of) the strings formed at the breaking of this anomalous $U(1)$ are local, in the sense that their energy per unit length can be localized in a finite region surrounding the string core, even though this energy is formally logarithmically infinite. It will be shown indeed that the axion field configuration can be made to

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wind around the strings so that any divergence must come from the region near the core instead of asymptotically. The cutoff scale that must then be introduced is thus a purely local quantity, definable in terms of the microscopic underlying fields and parameters. It is arguable that such a cutoff should be interpreted as the scale at which the effective model used throughout ceases to be valid.

This paper is divided into three parts; in the first one I expose the model which we use for studying string solutions. Then I discuss briefly the Higgs mechanism in this framework; in the final part I specifically give the local cosmic strings solutions and discuss some related phenomenological questions.

1. THE MODEL AND ITS RAISON D'ETRE

Compactification models for the heterotic string are known to lead in general to the presence in the 4-dimensional theory of a so-called universal axion field a [2]. At the supersymmetric level, this pseudoscalar field belongs to the same (chiral) supermultiplet S as the dilaton field s and they form a complex scalar field $s + ia$. The superfield S couples in a model-independent way to the gauge fields present in the theory; one has in particular in the Lagrangian

$$\mathcal{L} = -\frac{s}{4M_p} \sum_a F^a_{\mu\nu} F^a_{\mu\nu} + \frac{a}{4M_p} \sum_a F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} \tag{1}$$

where $F^a_{\mu\nu}$ is the field strength associated with the gauge field A^a_μ and M_P is the reduced Planck scale, the index a runs over all gauge groups, and

$$\tilde{F}^{a\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma} \tag{2}$$

An Abelian symmetry with gauge field A_μ may have (mixed) anomalies: under a gauge transformation of parameter α, the effective Lagrangian is no longer invariant, but picks up new terms (the anomaly) given by $\delta\mathcal{L} = -\frac{1}{2}\delta_{GS}\alpha \, \Sigma_a \, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$. This can be canceled by an apropriate shift of the axion a. Since there is a single model-independent axion (in the weakly coupled heterotic string spectrum), only one Abelian symmetry, henceforth referred to as $U(1)_X$, may be pseudo-anomalous.

One can write the supersymmetric Lagrangian of a model with such a pseudo-anomalous $U(1)_X$ symmetry:

$$\mathcal{L} = (\mathcal{K} + A_i^\dagger e^{X_i V} A_i)_{D\text{-term}} + \left(\frac{1}{4} S W^\alpha W_\alpha\right)_{F\text{-term}} + \text{h.c.} \tag{3}$$

Using the standard notations of ref. 24. S is the axion–dilaton superfield, V

is the gauge vector superfield, A_i are chiral superfields of respective charge X_i under the pseudo-anomalous $U(1)_X$ symmetry, and $\mathcal{K} = -\ln(S + \bar{S} - 4\delta_{GS} V)$ is the modified Kähler function for S [2]. The D-term of the Lagrangian is now invariant under the following transformations:

$$A_i \to e^{-q_i\Lambda} A_i \tag{4}$$

$$V \to V + i(\Lambda - \Lambda^\dagger) \tag{5}$$

$$S \to S + 4i\delta_{GS}\Lambda \tag{6}$$

where Λ is a chiral superfield which is the generalized gauge transformation parameter. The variation of the term $SW^\alpha W_\alpha$ under a restricted gauge transformation compensates for the 1-loop appearance of the gauge anomaly. A superpotential $\mathcal{W}(A_i)$ can also be added; one can show that it receives no contribution from S in perturbation theory (however, it may no longer be the case at the nonperturbative level). Integrating out the auxiliary fields, one obtains for the bosonic terms in the Lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{M_P^2}{4s^2}\partial^\mu s\partial_\mu s - (D_\mu\Phi_i)^\dagger(D^\mu\Phi_i) - \frac{1}{4}\frac{s}{M_P}F_{\mu\nu}F^{\mu\nu}\\ &+\frac{1}{4}\frac{a}{M_P}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{M_P^4}{s^2}\left(\frac{\partial^\mu a}{2M_P} - \delta_{GS}A^\mu\right)^2\\ &-\frac{M_P}{2s}\left(\frac{\delta_{GS}M_P^3}{s} + q_i\Phi_i^\dagger\Phi_i\right)^2\end{aligned} \tag{7}$$

where we have restored the Planck mass everywhere and have introduced the scalar fields Φ_i carrying the integer charge X_i (X_i has been rescaled by a factor 2) under the $U(1)_X$ symmetry, which are the lowest component of the chiral superfield A_i. The covariant derivative is defined by

$$D^\mu\Phi_i \equiv (\partial^\mu - iX_i A^\mu)\Phi_i \tag{8}$$

The δ_{GS} parameter (which fixes the scale of the symmetry breaking with respect to the fundamental scale of the theory, here given by M_P) may be computed in the framework of the weakly coupled string and is found to be [2]

$$\delta_{GS} = \frac{1}{192\pi^2}\sum_i X_i \tag{9}$$

where X_i are the charges of the different fields under $U(1)_X$.

We then obtain the model which we considered [1] by setting the dilaton to its VEV $\langle s\rangle = M_P/g^2$, which gives the gauge coupling constant of the $U(1)$ symmetry. Rescaling the Green Schwarz coefficient δ_{GS} and the axion by a factor g^2, one finds

$$\begin{aligned}\mathcal{L} = & -(D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) \\ & - \frac{1}{4g^2}\left(F_{\mu\nu} F^{\mu\nu} - \frac{a}{M_P} F^{\mu\nu} \tilde{F}_{\mu\nu}\right) \\ & - \delta_{GS}^2 M_P^2 A_\mu A^\mu + \delta_{GS} M_P A^\mu \partial_\mu a \\ & - \frac{1}{4} \partial_\mu a \, \partial^\mu a - V(\Phi_i) \end{aligned} \tag{10}$$

The potential $V(\Phi_i)$ is defined by

$$V(\Phi_i) \equiv \frac{g^2}{2} (\Phi_i^\dagger X_i \Phi_i + \delta_{GS} M_P^2)^2 \tag{11}$$

The Lagrangian (10) is now invariant under the following local gauge transformation with gauge parameter $\alpha(x^\mu)$:

$$\begin{aligned} \Phi_i &\to \Phi_i e^{iX_i\alpha} \\ A_\mu &\to A_\mu + \partial_\mu \alpha \\ a &\to a + 2M_P \delta_{GS} \alpha \end{aligned} \tag{12}$$

The transformation of the term $(a/4g^2 M_P) F^{\mu\nu} \tilde{F}_{\mu\nu}$ cancels the variation of the effective Lagrangian due to the anomaly, namely $\delta\mathcal{L} = -(1/2g^2)\delta_{GS}\, \alpha F^{\mu\nu} \tilde{F}_{\mu\nu}$ [assuming we are also transforming the fermions of the theory not written explicitly in (10)]. Making a rigid gauge transformation with parameter $\alpha = 2\pi$ without changing a as a first step, but transforming the other fields (including the fermions), leads us to interpret a as a periodic field of period $4\pi\delta_{GS} M_P$ through the redefinition $a \to a - 4\pi\delta_{GS} M_P$, which leaves the Lagrangian invariant. It is also manifest that a behaves like a phase, in the following rewriting of the kinetic term and of the axionic θ-term in $\mathcal{L}$:

$$\begin{aligned} \mathcal{L}_{\mathrm{kin},\theta} = & -\frac{1}{4g^2}\left(F^{\mu\nu} F_{\mu\nu} - \frac{a}{M_P} F^{\mu\nu} \tilde{F}_{\mu\nu}\right) \\ & - \partial^\mu \phi_i \, \partial_\mu \phi_i - \mathcal{D}^\mu \eta_i \, \mathcal{D}_\mu \eta_i - \mathcal{D}^\mu a \, \mathcal{D}_\mu a \end{aligned} \tag{13}$$

where we have defined $\Phi_i \equiv \phi_i e^{i\eta_i}$ (ϕ_i being the modulus of Φ_i) and

$$\mathcal{D}^\mu \eta_i = \phi_i X_i \left(\frac{\partial^\mu \eta_i}{X_i} - A^\mu\right) \tag{14}$$

$$\mathcal{D}^\mu a = M_P \, \delta_{GS} \left(\frac{\partial^\mu a}{2M_P \, \delta_{GS}} - A^\mu\right) \tag{15}$$

At this point of the discussion it should be noted that there are other sources of potential axions (axion meaning here a field associated with a Peccei–Quinn symmetry) in string theories than the universal one. These arise from zero modes of the antisymmetric tensor field $B_{\alpha\beta}$ which is present in the supergravity multiplet of the 10-dimensional theory. Components B_{ij} (where i, j are indices tangent to the six-dimensional compact space) can have axion-like couplings in a scheme-dependent way (in contrast to the universal axion which can be seen as coming from the components $B_{\mu\nu}$ tangent to the 4-dimensional noncompact space). In type I and II theories the scalars from the R–R sector are also potential axions. For the universal axion, as well as for the others, the exact pattern of symmetry one ends up with in the 4-dimensional theory is very scheme dependent since these fields receive mass from instanton effects (field theory, world sheet, or Dirichlet instantons) breaking the symmetry they are associated with. The universal axion, e.g., receives mass contributions from instantons of all the gauge groups which survive under the string scale, possibly including other groups than the standard model gauge groups. We did not include mass terms for the axion in our Lagrangian (7), (10) since they are very model dependent, and are suppressed by temperature in the early universe where cosmic strings are likely to form (they can, however, have an important effect when the temperature of the universe is decreasing). Moreover, one can look at the Lagrangian (10) as the most general one for an axion compensating some anomalous $U(1)_X$ symmetry, the couplings of the axion to the gauge fields in (10) being imposed by gauge invariance. In this latter case M_P has to be understood as the mass scale associated with the relevant theory.

2. HIGGS MECHANISM

Let us now work out the Higgs mechanism in this context. We consider for the sake of simplicity a single scalar field Φ of negative charge X and consequently we drop the i indices. $\mathcal{L}_{\text{kin},\theta}$ can be rewritten

$$\begin{aligned}\mathcal{L}_{\text{kin},\theta} = &-[M_P^2\,\delta_{GS}^2 + \phi^2 X^2]\\ &\times\left[A^\mu - \frac{\frac{1}{2}M_P\,\delta_{GS}\,\partial^\mu a + \phi^2 X\partial^\mu\,\eta}{M_P^2\,\delta_{GS}^2 + \phi^2 X^2}\right]^2\\ &-\frac{\phi^2 M_P^2\,\delta_{GS}^2\,X^2}{M_P^2\,\delta_{GS}^2 + \phi^2 X^2}\left[\frac{\partial^\mu a}{2M_P\,\delta_{GS}} - \frac{\partial^\mu\eta}{X}\right]^2\\ &+\frac{\delta_{GS}}{2g^2}\left(\frac{\phi^2 X^2}{M_P^2\,\delta_{GS}^2 + \phi^2\,X^2}\left[\frac{a}{2M_P\,\delta_{GS}} - \frac{\eta}{X}\right]\right.\end{aligned}$$

$$+\frac{\frac{1}{2}\, M_P\, \delta_{GS}\, a + \phi^2\, X\eta}{M_P^2\, \delta_{GS}^2 + \phi^2\, X^2}\Bigg) F_{\mu\nu}\, \tilde{F}^{\mu\nu} - \frac{1}{4g^2} F_{\mu\nu}\, F^{\mu\nu} \tag{16}$$

The linear combination appearing in this last equation,

$$\frac{a}{2M_P\delta_{GS}} - \frac{\eta}{X} \tag{17}$$

is the only gauge-invariant linear combination of η and a (up to a constant). The other one,

$$\ell \equiv \frac{\frac{1}{2} M_P\, \delta_{GS}\, a + \phi^2\, X\, \eta}{M_P^2\, \delta_{GS}^2 + \phi^2\, X^2} \tag{18}$$

has the property of being linearly independent of the previous one and of transforming under a gauge transformation (12) as $\ell \rightarrow \ell + \alpha$. We now assume explicitely that Φ takes its vacuum expectation value $\langle \Phi^\dagger \Phi \rangle \equiv \rho^2$ in order to minimize the potential (11):

$$\rho^2 = -\delta_{GS}\, M_P^2 / X \tag{19}$$

We are left with, among other fields, a massive scalar Higgs field corresponding to the modulus of Φ of mass m_x given by

$$m_x^2 = 2g^2\, \rho^2\, X^2 = -2\delta_{GS}\, X\, g^2\, M_P^2 \tag{20}$$

and we define

$$\hat{a} \equiv \left[\frac{a}{2M_P\, \delta_{GS}} - \frac{\eta}{X} \right] \frac{\sqrt{2}\rho M_P\, \delta_{GS}\, X}{\sqrt{M_P^2\, \delta_{GS}^2 + \rho^2\, X^2}} \tag{21}$$

and

$$F_a^2 = \frac{1}{128\pi^4} \frac{M_P^2\, g^4}{\rho^2\, X^2} (M_P^2\, \delta_{GS}^2 + \rho^2\, X^2)$$
$$= \frac{1}{128\pi^4} M_P^2\, g^4 \left[1 + \left(\frac{m_x}{M_P} \right)^2 \frac{1}{2g^2\, X^2} \right] \tag{22}$$

so that, with ρ being set,

$$\mathcal{L}_{\mathrm{kin},\theta} = -\, [M_P^2\, \delta_{GS}^2 + \rho^2\, X^2]\, [A^\mu - \partial^\mu \ell]^2 - \frac{1}{2}\, \partial^\mu\, \hat{a} \partial_\mu \hat{a}$$
$$+ \left[\frac{\hat{a}}{32\pi^2 F_a} + \frac{\delta_{GS}}{2g^2}\, \ell \right] F_{\mu\nu}\, \tilde{F}^{\mu\nu} - \frac{1}{4g^2}\, F_{\mu\nu}\, F^{\mu\nu} \tag{23}$$

we can now make a gauge transformation to cancel $\partial^\mu \ell$ by setting $\alpha = -\ell$. This leaves us with

$$\mathscr{L}_{\text{kin},\theta} = -\frac{m_A^2}{2g^2} A^\mu A_\mu - \frac{1}{2}\partial^\mu \hat{a}\, \partial_\mu \hat{a} + \frac{\hat{a}}{32\pi^2 F_a} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \tag{24}$$

where m_A given by

$$\begin{aligned} m_A^2 &= 2g^2 [\rho^2 X^2 + M_P^2 \delta_{GS}^2] \\ &= m_x^2 \left[1 + \left(\frac{m_x}{M_P}\right)^2 \frac{1}{2g^2 X^2} \right] \end{aligned} \tag{25}$$

is the mass of the gauge field after the symmetry breaking. The remaining symmetry,

$$\hat{a} \to \hat{a} + \frac{32\pi^2 F_a}{2g^2} \delta_{GS}\beta \tag{26}$$

is the rigid Peccei–Quinn symmetry, which compensates for the anomalous term arising from a rigid phase transformation of parameter β on the fermions.

To summarize, we have seen that in the presence of the axion the gauge boson of the pseudo-anomalous symmetry absorbs a linear combination ℓ of the axion and of the phase of the Higgs field. We are left with a rigid Peccei–Quinn symmetry, the remnant axion being the other linear combination $\hat{a}$ of the original string axion and of the phase of the Higgs field.

3. PSEUDO-ANOMALOUS $U(1)$ STRINGS

3.1. Cosmic String Solutions

We now look for stationary local cosmic string solutions of the field equations derived from Eq. (10) provided the underlying $U(1)$ symmetry is indeed broken, which implies that at least one of the eigenvalues X_i is negative. So in this section we shall assume again only one field Φ with charge X, with $X < 0$. The field equations are

$$\Box a = 2\delta_{GS} M_P\, \partial_\mu A^\mu - \frac{1}{2g^2 M_P} F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{27}$$

$$\Box \phi = \phi(\partial_\mu \eta - X A_\mu)^2 + g^2 X \phi (X \phi^2 + \delta_{GS} M_P^2) \tag{28}$$

$$\partial_\mu [\phi^2 (\partial^\mu \eta - X A^\mu)] = 0 \tag{29}$$

$$\frac{1}{g^2}\partial_\mu\left(\frac{a}{M_P}\tilde{F}_{\mu\nu} - F^{\mu\nu}\right) = \delta_{GS}M_P\partial^\nu a - 2\delta_{GS}^2M_P^2A^\nu + 2X\phi^2(\partial^\nu\eta - XA^\nu) \quad (30)$$

Using Eq. (2), which implies $\partial_\mu \tilde{F}^{\mu\nu} = 0$, and deriving Eq. (30) with respect to x^ν, We obtain, with the help of Eqs. (27) and (29),

$$F_{\mu\nu}\,\tilde{F}^{\mu\nu} = 0 \quad (31)$$

and we can rewrite Eq. (30),

$$\frac{1}{g^2}\partial_\mu\, F^{\mu\nu} = \frac{1}{M_P}\tilde{F}^{\mu\nu}\,\partial_\mu\, a + \mathcal{J}^\nu + J^\nu \quad (32)$$

where the currents are defined as

$$J^\mu = -2X\,\phi^2\,(\partial^\mu\eta - X\,A^\mu) = -2X\,\phi\,\mathcal{D}^\mu\eta \quad (33)$$

and

$$\mathcal{J}^\mu = -\delta_{GS}\,M_P\,(\partial^\mu\,a - 2\delta_{GS}\,M_P\,A^\mu) = -2M_P\,\delta_{GS}\,\mathcal{D}^\mu\,a \quad (34)$$

Equations (27) and (29) then simply express the two current conservations $\partial\cdot J = \partial\cdot\mathcal{J} = 0$, when account is taken of Eq. (31).

Looking for a local cosmic string solution, we ask that on a 1-circle at infinity on a 2-plane transverse to the string, the kinetic and potential energy of the different fields vanish. Namely,

$$F^{\mu\nu}\,F_{\mu\nu} = 0 \quad (35)$$

$$(D^\mu\,\Phi)^2 = 0 \quad (36)$$

$$(\mathcal{D}^\mu\,a)^2 = 0 \quad (37)$$

$$V(\Phi) = 0 \quad (38)$$

As usual the fact that the Π_1 of the vacuum manifold derived from the potential (11) is nontrivial leads to the possibility to have a nontrivial winding solution of Eq. (38) with an asymptotic behavior; in cylindrical coordinates,

$$\Phi = \rho e^{i\eta} \quad (39)$$

$$\eta = n\theta \quad (40)$$

where ρ is defined in (19) and n is the string winding number. Equations (36) and (35) can then be satisfied asymptotically by taking A_μ a pure gauge and in such a way as to compensate for the Higgs field energy density:

$$A_\mu = \partial_\mu\,\eta/X \quad (41)$$

as in the Nielsen–Olesen [11] solution. Equation (37) then induces a winding of the axion field with a winding number related to that of η by

$$a = \frac{2\delta_{GS} M_P}{X} \eta \tag{42}$$

a perfectly legitimate choice, as it should be remembered that a is a periodic field of period $4\pi\delta_{GS}M_P$ [$|X|$ is here implicitly equal to 1, but it is clear that for any other value a solution such as (42) exists].

The energy of the cosmic string configuration is confined in the string as in the Nielsen–Olesen strings (and for the very same reason) and the cosmic string is perfectly local. This is in striking contrast with the case of a global string, where a divergent behavior of the energy density arises because the energy is not localized and a large-distance cutoff must be introduced. In this case, a divergence is still to be found, as we will now see, but this time at a small distance near string core so that the total energy is localized in a finite region of space.

The stress-energy tensor is given by

$$T^{\mu}_{\nu} = -2g^{\mu\gamma} \frac{\delta \mathcal{L}}{\delta g^{\gamma\nu}} + \delta^{\mu}_{\nu} \mathcal{L} \tag{43}$$

which reads explicitly

$$T^{\mu\nu} = 2[\partial^{\mu} \phi \partial^{\nu} \phi - \frac{1}{2} g^{\mu\nu} (\partial\phi)^2] + \frac{1}{g^2} \left(F^{\rho\mu} F^{\nu}_{\rho} - \frac{1}{4} g^{\mu\nu} F \cdot F \right) \tag{44}$$

$$- \frac{1}{2} g^2 g^{\mu\nu} (X \phi^2 + \delta_{GS} M_P^2)^2 + \frac{1}{2\delta_{GS}^2 M_P^2} [\mathcal{J}^{\mu} \mathcal{J}^{\nu} - \frac{1}{2} g^{\mu\nu} \mathcal{J}^2] + \frac{1}{2X^2 \phi^2} [J^{\mu} J^{\nu} - \frac{1}{2} g^{\mu\nu} J^2] \tag{45}$$

where account has been taken of the field equations. The energy per unit length U and tension T will then be defined respectively as

$$U = \int d\theta \, r \, dr \, T^{tt} \qquad \text{and} \qquad T = -\int d\theta \, r \, dr \, T^{zz} \tag{46}$$

The question as to whether the corresponding string solution is local or global is then equivalent to asking whether these quantities are asymptotically convergent (i.e., at large distances). The total energy per unit length (and tension) is not finite, however, in this simple string model, for it contains the term

$$U = \text{f.p.} + 2\pi \int \frac{dr}{r} \left(\frac{\delta_{GS} M_P n}{X} - \delta_{GS} M_P A_\theta \right)^2 \qquad (47)$$

(f.p. denotes the finite part of the integral), so that, since A_θ must vanish by symmetry in the string core, one ends up with

$$U = \text{f.p.} + 2\pi \left(\frac{\delta_{GS} M_P n}{X} \right)^2 \ln \left(\frac{R_A}{r_a} \right) \qquad (48)$$

where R_A is the radius at which A_μ reaches its asymptotic behavior, i.e., roughly its Compton wavelength m_A given in (25), while r_a is defined as the radius at which the effective field theory (10) ceases to be valid, presumably of order M_P^{-1}; the correction factor is thus expected to be of order unity for most theories. Hence, as claimed, the strings in this model can be made local with a logarithmically divergent energy. The regularization scale r_a is, however, a short-distance cutoff, solely dependent on the microscopic structure, and involves neither the interstring distance nor the string curvature radius. In particular, the gravitational properties of the corresponding strings are those of a usual Kibble–Vilenkin string [13], given the equation of state is that of the Goto–Nambu string $U = T =$ const, and the light deflection is independent of the impact parameter [14].

The solution (42) turns out, as can be explicitly checked using Eqs. (27) and (31), to be the only possible nontrivial and asymptotically converging solution.

Moreover, the stationnary solution (42) shows the axion gradient to be orthogonal to $\tilde{F}^{\mu\nu}$, i.e., $\partial_\mu a \tilde{F}^{\mu\nu} = 0$. Therefore, Eqs. (27)–(32) reduce to the usual Nielsen–Olesen set of equations [11], with the axion coupling using the string solution as a source term. It is therefore not surprising that the resulting string turns out to be local.

The local string solution we have found can also be considered using the new dynamical variables $\hat{a}$ and l defined in the previous section. In this language the local strings considered are the one obtained by a winding of l and A_μ around the string core, whereas $\hat{a}$ is not winding. A winding of $\hat{a}$ would generate global axionic-like strings decoupled from the previous ones.

3.2. Local String Genesis

Forming cosmic strings during a phase transition is a very complicated problem involving thermal and quantum phase fluctuations [15]. It is far from clear how a and η fluctuations will be correlated (even though they presumably will be). One can consider, as a toy model, the possibility that a network of two different kinds of strings will be formed right after the phase transition, call them a-strings and η-strings, with the meaning that an a-string is generated whenever the axion field winds (ordinary axion string), while an η-string appears when the Higgs field Φ winds. Both kinds of strings are initially global since for both of them, only part of the covariant derivatives can be made to vanish. We expect, however, the string network to consist, after some time, of only these local strings together with the usual global axionic strings.

Let us consider an axionic string with no Higgs winding: as $A^\mu \neq 0$, the vacuum solution $\Phi = \rho$ [Eq. (19)] is not a solution, and thus the axionic string field configuration is unstable. As a result of Eq. (28), the Higgs field amplitude tends to vanish in the string core. At this point, it becomes, near the core, topologically possible for its phase to start winding around the string, which it will do since this minimizes the total energy while satisfying the topological requirement that A_μ flux be quantized. Such a winding will propagate away from the string.

Conversely, consider the stability of an η-string with $a = 0$. The conservation of $\mathcal{T}$ implies, as one can fix $\partial_\mu A^\mu = 0$, that $\Box a = 0$, whose general time-dependent solution is $a = a(|\mathbf{r}| \pm t)$. Given the cylindrical symmetry, this solution can be further separated into $a = f(r - t)\theta$. This means that having a winding of a that sets up propagating away from the string is among the solutions. As this configuration ultimately would minimize the total energy, provided $\lim_{t\to\infty} f = -2\delta_{GS} M_P / X$, this means that the original string is again unstable and will evolve into the stationary solution that we derived in the previous section.

It should be remarked at this point that this time evolution can in fact only be accelerated when one takes into account the coupling between a and η: if either one of them is winding, then the other one will exhibit a tendency to also wind, in order to minimize locally the energy density. Indeed, it is not even really clear whether the string configurations we started with would even be present at the string-forming phase transition. What is clear, however, is that after some time, all the string network would consist of local strings having no long-distance interactions. This means in particular that the relevant scale, if no inflationary period is to occur after the string formations, should not exceed the GUT scale, in order to avoid cosmological contradictions.

3.3. Constraints on the Scale of the Symmetry Breaking

The cosmological evolution of the network of strings formed in these theories may also lead to serious constraints on the Green–Schwarz coefficient. If domain walls form connecting the strings, which itself depends on the (temperature-dependent) potential generated by instantons, the network is known to rapidly (i.e., in less than a Hubble time) decay into massive radiation and the usual constraint relative to the axion mass would hold [7, 8, 16]. If, however, the string network is considered essentially stable, then its impact on the microwave background limits the symmetry-breaking scale $\delta_{GS} M_P$ through the observational requirement that the temperature fluctuations be not too large [17], i.e.,

$$GU \lesssim 10^{-6}$$

with G the Newton constant, $G = M_P^{-2}/(8\pi^2)$. Therefore, the cosmological constraint reads

$$\delta_{GS} \lesssim 10^{-2} \tag{49}$$

a very restrictive constraint indeed, which can be compared with the scale given in (9) or with similar predictions in the strongly coupled heterotic strings [20].

The strings that we have discussed here might appear in connection with a scenario of inflation. Indeed, the potential (11) is used for inflation in the scenario known as D-term inflation [18]: inflation takes place in a direction neutral under $U(1)$ and the corresponding vacuum energy is simply given by

$$V_0 = \frac{1}{2} g^2 \delta_{GS}^2 M_P^4 \tag{50}$$

The $U(1)$-breaking minimum is reached after inflation, which leads to cosmic string formation. Such an inflation era cannot therefore dilute the density of cosmic strings and one must study a mixed scenario [9]. It is interesting to note that, under the assumption that microwave background anisotropies are predominantly produced by inflation, the experimental data put a contraint [5, 10, 19] on the scale $\xi \equiv \delta_{GS}^{1/2} M_P$, which is stronger than (49). Several ways have been proposed [5, 19] in order to lower this scale. They would at the same time ease the constraint (49).

3.4. String Currents

It is clear that the model (10) is no longer supersymmetric, since we have set the dilaton to its VEV, whereas the axion is a dynamical field and belongs to the same multiplet. Allowing the dilaton to be dynamic as in (7), we can easily preserve supersymmetry after the breaking of the $U(1)$, which can be desirable if we assume that supersymmetry is broken at some lower scale than the one of $U(1)$ breaking. Since the D-term, which gives the potential (11), is zero after symmetry breaking, one has only to be sure that the VEV taken by the Higgs field Φ does not destabilize the vanishing energy of the vacuum through the superpotential (an explicit example is given in the first of refs. 2). Following Hughes and Polchinski [3], we expect that, even when the Lagrangian is fully supersymmetric, at least some of the supersymmetry generators will be broken by the cosmic string configuration because this configuration breaks translational invariance. This will lead to goldstinos, which are massless Fermi fields on the string arising from Fermi zero modes in the underlying theory and give rise to supercurrents [12] (some explicit example of these superconducting supersymmetric cosmic strings have recently been worken out by Davis *et al.* [22, 23]). These currents, which can also appear in a nonsupersymmetric model from the coupling of the string to other fields, fermionic in particular, tend to raise the stress-energy tensor degeneracy in such a way that the energy per unit length and tension become dynamical variables. For loop solutions, this means a whole new class of equilibrium solutions, named vortons, whose stability would imply a cosmological catastrophe [21]. If these objects were to form, (49) would change into a drastically stronger constraint. Issues such as the explicit construction of the currents, their relation to supersymmetry, and whether supersymmetry breaking might destabilize the currents [22], thereby effectively curing the model from the vorton problem, still deserve investigation; as do the possible consequences of having a dynamical dilaton [26]. We are also planning to look at a more realistic model in the framework of horizontal symmetries.

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